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Modeling Communication Protocols for Client-Server Networks: Model Checking Dynamical Systems

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Abstract. Certain class of communication protocols is well modeled by dynamical systems on torus-like spaces. If *m* agents/processes communicate/cooperate in a system then it can be modeled by a dynamical system on *m*-dimensional torus-like space. No mathematical machinery is known for analysis of these models for m>3. In this paper we suggest a method of investigation of certain properties of these systems using their symbolic dynamics. We construct a marked graph such that all symbolic trajectories of dynamical system under consideration are runs on the marked graph. Therefore, we show that some properties of a dynamic system can be proved as follows: formulate a property in a language of temporal logic (or program logic); model check this formula in the constructed marked graph using it as a Kripke structure.

Keywords: Communication Systems and Networks, Client-Server Systems, Dynamical Systems, Model Checking.

1. Introduction

Consider a distributed system consisting of *n* clients and *k* servers such that each client has its own program that is independent from other clients. According to its program a client processes data and makes requests to shared servers. Concurrent requests from a client to several servers are not allowed. Moreover, a client cannot proceed to process data until it receives a response to its request. For all clients we assume that:

- Programs are non-terminating loops represented by a thread of control C_i is $C_i^1 S_i^1 C_i^2 S_i^2 \ldots C_i^m S_i^m C_i^1 S_i^1 C_i^2 S_i^2 \ldots$ where C_i^j is the *j*'th fragment of a program of *i*'th client, S_i^j is a request to a server $r=S(i,j)$;
- Given first execution of a program fragment C_i^j took time t_i^j then all subsequent executions of this fragment will always take time t_i^j. Same applies to processing of server requests if no waiting occurs. Apparently, after *m* steps the sequence repeat itself;
- All clients are assumed to work concurrently.

Each server has a thread dedicated to a client. It can execute one thread at a time. Therefore, servers maximal queue size is n. Each server has its own algorithm of queue service determined by the communication protocol. Most commonly used protocols result in FIFO service.

For a model with just one client C_1 and arbitrary many servers there is no waiting regardless of the times required for execution of program fragments and requests to servers. In general the possibility of bottlenecking exist in any of *k* servers to the extent when the system is deadlocked and the only way out is to drop requests [BS, 90].

Provided algorithms of queue service are defined for all servers we have a class of dynamic systems parameterized by the times of executions of program fragments and server requests. In this paper we are interested in properties of the steady state behavior of all systems belonging to a given class. Examples of such as properties are: every system in this class degenerates to its subspace, the system is bottlenecked in a particular server, etc. We introduce the method to decide if properties that can be expressed in terms of symbolic dynamics of any system in the class hold for all systems in this class. The method involves the following steps:

1. Formalization of a property of interest;
2. Construction of a marked graph from a dynamical system ;
3. Model-checking of a property.

Steps 1 and 3 have been thoroughly investigated in recent years and are fully automated (see [CS, 00]). We address step 2 in this paper.

The outline of the paper is as follows: in section 2 we formally define communication systems of interest by means of queuing systems. In section 3 we redefine the class of dynamical systems under consideration in terms

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of admissible state spaces and vector bundles. The new approach automatically define a class of dynamical system in process of definition of a specific dynamical system. Definition of symbolic dynamics on such a space come as a natural consequence of the new definition of dynamical system. In section 4 we show the algorithm to construct marked graph such that if a trajectory is realizable in any system of the class then its symbolic trace is a run on the aforementioned marked graph. Therefore, if a property holds in all systems then its negation must fail in a constructed graph when it is used as a model for a property. In section 5 we discuss our results.

2. Definitions and notation

We consider a class of closed queuing networks in which every network consist of n clients and k servers denoted $C_1, \dots, C_n$ and $S_1, \dots, S_k$ correspondingly (see Fig. 2.1). In such queuing network each client generate a call that has its circular route in the network, i.e. i'th client generate a call with the route $St_i(1), \dots, St_i(l_i)$ where $St_i(j) \in \{C_i, S_1, \dots, S_k\}$ is j'th step in the service of i'th call. The route $St_i(1), \dots, St_i(l_i)$ is such that:

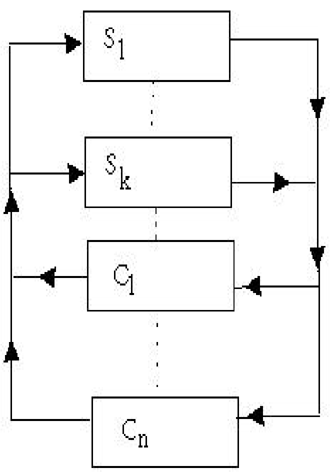


Fig. 2.1.

- if the step j of i'th call is served in the client ($St_i(j)=C_i$) then the step $j+1$ is served in a server ($St_i(j+1)) \in \{S_1, \dots, S_k\}$),
- if the step j of i'th call is served in a server ($St_i(j+1)) \in \{S_1, \dots, S_k\}$) then the step $j+1$ is served in the client ($St_i(j)=C_i$) ,
- the next step to be served after the step l_i is the step 1.

We assume that the service time of i'th call in j'th step of the route $T(i,j)$ is the same for any cycle and is constant. A server S_j has a queue of a maximal size n while clients C_i have no queue. For n=1 there is no waiting and a call is always worked on. The dynamics of such system given by the transition function that maps current state and time into a new state at the time is known as a standard flow on a circumference $f: C^1 \times R \to C^1 :: f(x,t) = [x+t]_{\mathrm{mod} \Sigma_j^{l_i} T(i,j)}$ (see Fig. 2.2). For n>1 a specific protocol must be defined to determine the service discipline in servers. If each server can serve simultaneously all n calls without slowing down then the model results in no waiting again. In this case the dynamics of the system is a product of individual dynamics of calls and is given by

$<f_1, \dots, f_n>: \boldsymbol{T}^n \times R \to \boldsymbol{T}^n :: <f_1, \dots, f_n>(\mathbf{x},t) = [f_1(x_1,t), \dots, f_n(x_n,t)]$ where $\boldsymbol{T}^n$ is an n-dimensional torus. This model is usually referred as standard dynamics on torus. In [WHE, 96] communication protocols resulting in FCFS and LCFS service discipline in the servers were considered. In [Wol, 94] we considered multiprocessor systems that lead to similar queuing networks and dynamical systems. Yet in all these cases dynamics was defined on torus-like spaces that were received as a result of surgery performed on n-dimensional torai. In this paper the 2-dimensional LFCS system (Fig. 2.3) is used as a main example. This system consist of one server and two clients. A job is processed first by a client and then send to a sever. If at some point in time a server has more than one request to process then it chooses requests in LCFS order. Therefore the route for i'th client is C_i, S_i with times T_{C_i}, T_{S_i} correspondingly. The state space of the system is constructed out of a two dimensional marked torus $\mathbf{T^2}$ with the base circumferences of the length $T_{C_i}+T_{S_i}$, i=0,1. Before performing surgery on the $\mathbf{T^2}$ it is marked into rectangles

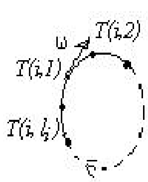


Fig. 2.2.

$D_{00} = T_{C_0} \times T_{C_1}$, $D_{01} = T_{C_0} \times T_{S_1}$, $D_{10} = T_{S_0} \times T_{C_1}$, $D_{11} = T_{S_0} \times T_{S_1}$. The rectangle D_{11} is cut out of the torus and then covered twice by rectangles

D_{11}^0, D_{11}^1. The dynamics on D_{00}, D_{10}, D_{01} is the standard flow on the straight line with vector field (1,1) while vector field on D_{11}^0 is (1,0) corresponding serving client 0 first, and on D_{11}^1 is (0,1) corresponding to serving client 1 first.

3. Cubic topological spaces and dynamics consistent with cubic structure

3.1 Cubic Spaces. Let us first define a standard n-dimensional cube C^n Î R^n. Following similar definition of the standard n-dimensional simplex (see for example [Spa,95]), an n-dimensional cube

$$C^n=\{(x_0,x_1,...,x_{n-1})\in R^n|0\leq x_i\leq 1\}$$

is defined as a product of n unit intervals I. A point of the form e_i such that $x_i\in\{0,1\}$ for all i is called a corner of the cube. In other words e_i=**bin**(i) for some $i\in\{0,1,...,2^n\}$. Here **bin**(i) is binary representation of i of length n. For each subset P Ì $[n-1]$ (where $[n-1]=\{0,1,...,n-1\}$) there exist $2^{n-|P|}$ P-faces of C^n. Each P-face is of the form:

$$\{(x_0,x_1,...,x_{n-1})\in R^n|0\leq x_i\leq 1 \text{ for all } i\in P,\ x_i\in\{0,1\} \text{ for all } i\notin P\}.$$

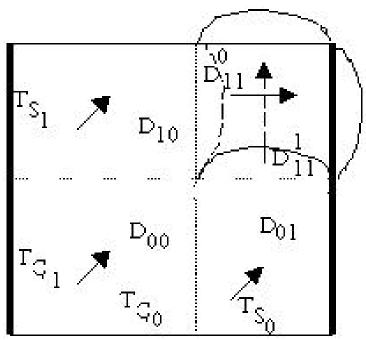


Fig. 2.3

For each inclusion

$$i:[k]\rightarrow[k+1]::j\rightarrow\begin{cases} i \text{ if } i<j \\ i+1 \text{ otherwise}\end{cases}$$

two face maps are be defined in obvious way:

$$\partial_i^j(k):C^{k-1}\rightarrow C^k::(x_0,x_1,...,x_i,...,x_{k-1})\rightarrow(x_0,x_1,...,x_i,j,x_{i+1},...,x_k)$$

where $j\in\{0,1\}$. Once the face maps are defined it is more convenient to determine faces of arbitrary dimensions by giving an increasing map f:[m]→[n] that has an image of P instead of the subset P Ì $[n-1]$, where $card[P]=m+1$. Obviously for each f there exist exactly 2^{n-m} linear maps that map C^m into C^n preserving the order of corners in the image. One such map exist for each $i\in\{0,1,...,2^{n-m}-1\}$. Let **bin**($i$)=$x_0x_1...x_{n-m}$, then there is unique linear map

$$C(f,x_0x_1\ldots x_{n-m})=\partial_i^{x_0}(n-1)\cdot\ldots\cdot\partial_i^{x_{n-m}}(m):C^m\rightarrow C^n$$

that preserve order of corners and the image of which is P-face.

As one can expect cubic spaces are defined in the manner similar to triangulated spaces. In particular unorthodox definition of triangular spaces in [GM,97] is modified below to define cubic spaces. The following structure is called the sewing data:

- The data set is a calibrated set $X=\cup_{i=0}^n X_i$. It defines the set of cubes to be sewed, i.e., the set X_o of points, the set X_1 of unit intervals, the set X_2 of unit squares, ..., the set X_n of n-dimensional cubes, ... Here elements of X_i are indices enumerating cubes of dimension i.
- The sewing method is set of maps of the form X_n®X_m. One map is given for every pair (I,**bin**(i)) such that $I\subset[n]$, card(I)=m and $i\in\{0,1,...,2^{n-m}-1\}$. Informally, such map shows which m-dimensional cube is identified with (I,**bin**(i))-face of corresponding n-dimensional cube. More precisely, let a face be given by a map f:[m]→[n] and **bin**(i)=$x_0x_1...x_{n-m}$. Let $X(f,$**bin**$(i)):X_n$®X_m be corresponding sewing map. Then the set $\cup_{n,m}\{X(f,$**bin**$(i))\}_n^m$ must satisfy the following conditions:
  1. X(id,0)=id;
  2. $X(g^{\circ}f,$**bin**(k)*$\mathbf{bin}$(s))=$X(f,$**bin**(k))· $X(g,$**bin**(s)) where **bin**(k)***bin**(s) is a concatenation of binary sequences.

  In other words these conditions mean that cubes of the same dimension cannot be identified and that 'a face of a cubs face is a face of the cube'.

Topological space $|X|=\sqcup_{i=0}^{\infty}(X_i\times C^i)/\boldsymbol{R}$ is the result of sewing using the sewing data defined above. Here $\boldsymbol{R}$ is the minimal equivalence relation that identifies the points $(x,s)\in(X_m\times C^m)$ and $(y,t)\in(X_n\times C^n)$ when for a pair <f,**bin**(i)> the following holds:

$$x=X(f,x_0x_1...x_{n-m})(y),\quad t=C(f,x_0x_1...x_{n-m})(s).$$

Here f:[m]→[n] and $i\in\{0,1,...,2^{n-m}-1\}$, **bin**($i$)=$x_0x_1...x_{n-m}$. We denote this relation

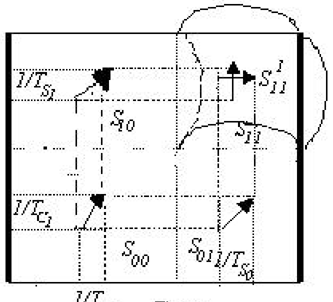


Fig. 3.2

$$(x,s)\sim^{< X(f,x_0x_1...x_{n-m}),C(f,x_0x_1\ldots x_{n-m})>}(y,t).$$

Canonical topology on |X| is the weakest topology in which the factorization map

$$\pi{:}|\_|_{i=0}^{\infty}(X_i\times C^i)\rightarrow|\_|_{i=0}^{\infty}(X_i\times C^i)/\boldsymbol{R}$$

is continuous. The space |X| together with the sewing data is called a cubic space and the sewing data for |X| is also called its cubic structure.

Let us denote $\underline{C}^n$ the interior of C^n for n≥0 (note that $\underline{C}^0=C^0$). Consider a cubic structure $[X_i,X(f,\mathbf{bin}(k))]$ and the corresponding canonical map $\pi|\_|_{i=0}^{\infty}(X_i\times C^i)\rightarrow|X|$. The latter induce a map

$$\underline{\pi}{:}|\_|_{i=0}^{\infty}(X_i\times\underline{C}^i)\rightarrow|X|.$$

Proposition 3.1.1. The map $\underline{\pi}$ is a set theoretic bijection.

Proof outline: The surjectivity of $\underline{\pi}$ is straightforward. Let us assume that (x,s) and (x',s') from $|\_|_{i=0}^{\infty}(X_i\times\underline{C}^i)$ are projected into the same point in |X|. It is possible only if both x and x' belong to the same X_k. The sequence of equivalence pairs must contain only x_{l_i}, $l_i>k$. Then we can shorten the sequence of equivalence pairs by combining equivalences using 3.1.a.. Continuing that way we come to $(x,s)\sim^{< X(f,x_0x_1...x_{n-m}),C(f,x_0x_1\ldots x_{n-m})>}(x',s')$. But this is possible only if $(x,s)=(x',s')$.☐

3.2. Examples of cubic spaces. a. An n-dimensional cube $\boldsymbol{I}^n$. In this case X_i={a number of subsets of n of cardinality i}×{0,1,...2^{n-i}-1}. For $f{:}[i]\rightarrow[j]$ and $k\in\{0,1,...,2^{j-i}-1\}$, $\mathbf{bin}(k)=x_0x_1...x_{j-i}$ the map $X(f,x_0x_1...x_{j-i})$ translates a pair (g,s) where $g{:}[j]\rightarrow[n]$, $s\in\{0,1,...,2^{n-j}-1\}$ into $(g{\cdot}f,\mathbf{bin}(s)*\mathbf{bin}(k))$. In other words, the n-dimensional cube is scattered into its faces and then sewed together anew. Obviously, here cardinality of each X_i is:

$$card(X)=\binom{n}{i}\quad ☐$$

b. 2-dimensional LIFO space. The sewing data is defined as follows:
The data set: $X_0=\{e_{00},e_{01},e_{10},e_{11}\}$, $X_1=\{x_{00},x_{01},x_{01}^1,x_{10},x_{11},x_{11}^1\ y_{00},y_{01},y_{01}^1,y_{10},y_{11},y_{11}^1\}$, $X_2=\{S_{00},S_{01},S_{10},S_{11}^0,S_{11}^1\}$;
The sewing methnod: for unique $f{:}[0]\rightarrow[1]$ the $X(f,0)(x_{00})=e_{00}$, $X(f,1)(x_{00})=e_{10}$, $X(f,0)(x_{01})=e_{10}$, $X(f,1)(x_{01})=e_{00}$, $X(f,0)(x_{01}^1)=e_{10}$, $X(f,1)(x_{01}^1)=e_{00}$, $X(f,0)(x_{10})=e_{01}$, $X(f,1)(x_{10})=e_{11}$, $X(f,0)(x_{11})=e_{11}$, $X(f,1)(x_{11})=e_{01}$, $X(f,0)(x_{11}^1)=e_{11}$, $X(f,1)(x_{11}^1)=e_{01}$, $X(f,0)(y_{00})=e_{00}$, $X(f,1)(y_{00})=e_{01}$, $X(f,0)(y_{01})=e_{01}$, $X(f,1)(y_{01})=e_{00}$, $X(f,0)(y_{01}^1)=e_{01}$, $X(f,1)(y_{01}^1)=e_{00}$, $X(f,0)(y_{10})=e_{10}$, $X(f,1)(y_{10})=e_{11}$, $X(f,0)(y_{11})=e_{11}$, $X(f,1)(y_{11})=e_{10}$, $X(f,0)(y_{11}^1)=e_{11}$, $X(f,1)(y_{11}^1)=e_{10}$;
Denote $f{:}[1]\rightarrow[2]{::}0\rightarrow0$ and $g{:}[1]\rightarrow[2]{::}0\rightarrow1$. Then $X(f,0)(S_{oo})=x_{00}$, $X(f,1)(S_{oo})=x_{10}$, $X(g,0)(S_{oo})=y_{00}$, $X(g,1)(S_{oo})=y_{10}$, $X(f,0)(S_{o1})=x_{01}$, $X(f,1)(S_{o1})=x_{11}$, $X(g,0)(S_{o1})=y_{10}$, $X(g,1)(S_{o1})=y_{00}$, $X(f,0)(S_{10})=x_{10}$, $X(f,1)(S_{10})=x_{00}$, $X(g,0)(S_{10})=y_{01}$, $X(g,1)(S_{10})=y_{11}$, $X(f,0)(S_{11})=x_{11}^1$, $X(f,1)(S_{11})=x_{01}^1$, $X(g,0)(S_{11})=y_{11}$, $X(g,1)(S_{11})=y_{01}$, $X(f,0)(S_{11}^1)=x_{11}$, $X(f,1)(S_{11}^1)=x_{01}$, $X(g,0)(S_{11}^1)=y_{11}^1$, $X(g,1)(S_{11})=y_{01}^1$. Obviously, the above sewing data defines the cubic space that is LIFO state space (see fig. 3.1.).☐

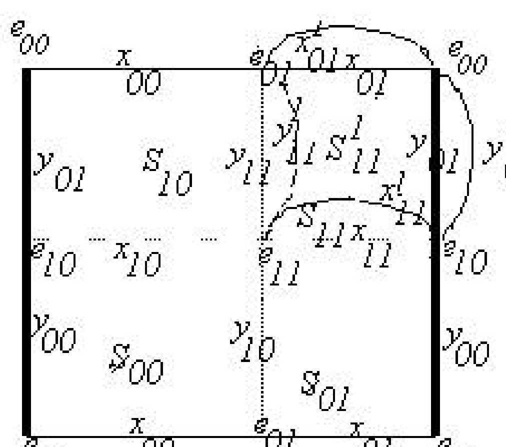


Fig. 3.1.

3.3. Dynamics on cubic spaces consistent with cubic structure. Let |X| be a cubic space, $[X_i,X(f,\mathbf{bin}(k))]$ be the corresponding cubic structure with the maximal nonempty index being k (X_k¹Øand for all $i>k$, $X_i=\varnothing$). Define a dynamical system on |X| by supplying a vector bundle determined at any point $y_0\in|X|$ by a set of standard equations of the form

$$dx_i/dt|_{y_0}=f_i(x_o,x_1,\ldots,x_k)\qquad(3.3.1.)$$

To be consistent with a cubic structure one should be able to define a vector field on $|\_|_{i=0}^{\infty}(X_i\times C^i)$ in such a way that factorization by $\boldsymbol{R}$ result in correctly defined flow on |X|. Moreover, the flow should be consistent with face maps in the sense of the following ‘exclusive or’ condition.

Requirement 3.3.2. One of the conditions below hold:

1. the flow is transverse to faces in coordinate *i*. It 'leaves' an initial *i*-face of *k*-dimensional cube and enters 'final' i-face of the cube, i.e., in *k*-dimensional cube *s* for every $y \hat{I}$ $<X(f,0),\partial_i^0(k)>(s\times C^k)$ holds $dx_i/dt)|_y = f_I(x_o,x_1,\ldots,x_k)>0$ and for every $y_0\in<X(f,1),\partial_i^1(k)>(s\times C^k)$ holds $dx_i/dt|_{y-0}=f_I(x_o,x_1,\ldots,x_k)>0$;
2. The flow in the cubs' interior is collinear with coordinate *i* faces and within *i*-faces condition 3.3.2.a. holds. This means that 3.3.2.a is applied to $<X(f,0),\partial_i^0(k)>(s\times C^k)$ and $<X(f,1),\partial_i^1(k)>(s\times C^k)$ of dimension $k:=k-1$.

The flow should have no fixed points in the interior of cubes of maximal dimensionality. Moreover, inside the interior of these cubes it must be smooth enough to ensure existence of solutions for 3.3.1. Standard approach (see for example [Arn,89]) is to require $d\mathbf{x}/dt=\mathbf{f}(\mathbf{x})$ to be continuously differentiable in every point of maximal cubes' interior. However, to have more realistic models limited discontinuities on the faces may be allowed, i.e.

$$»dx_i/dt|_y - dx_i/dt|_{y-0}»<\infty$$

3.4. Example 3.2.b. (continued) We can define LIFO dynamical system by setting for any $y \hat{I} S_{00}$ vector field to

$$dx_0/dt=1/T_{C_0};\quad dx_0/dt=1/T_{C_1};$$

for any $y \hat{I} S_{01}$ vector field to

$$dx_0/dt=1/T_{S_0};\quad dx_0/dt=1/T_{C_1};$$

for any $y \hat{I} S_{10}$ vector field to

$$dx_0/dt=1/T_{C_0};\quad dx_0/dt=1/T_{S_1};$$

for any $y \hat{I} S_{11}$ vector field to

$$dx_0/dt=1/T_{S_0};\quad dx_0/dt=0;$$

for any $y \hat{I} S_{11}^1$ vector field to

$$dx_0/dt=0;\quad dx_0/dt=1/T_{S_1}.$$

The rest of the vector field is uniquely expanded in conformance with 3.3.2. Note that the defined system is not the same as the LIFO example from section 2, but it is possible to show that

Claim 3.4.1. The dynamical system defined on cubic space 3.2.b. consistently with its cubic structure is homeomorphic to LIFO dynamical system from [WHE, 98].

Proof is omitted because of space limitations. ☐

3. 5. R^n- subsets with cubic structure. Let $[X_i,X(f,\mathbf{bin}(k))]$ be a cubic structure of cubic space $|X|$, let *m* be its maximal nonempty grading. ($X_m {}^1\varnothing$, for all $k>m$ $X_k=\varnothing$). So far we did not limit the way cubic spaces are sewed. Therefore, for a sewing data $[X_i,X(f,\mathbf{bin}(k))]_0^n$ it is possible that there exist a pair of elements of some grading *i* such that l'th face of the first element of the pair is the k-th face of the second element of the pair, i.e., for some *i* there exist $a,b\in X_i$ such that for some $f:[i-1]\to[i]$, $g: [i-1]\to[i]$, $g {}^1 f$, and $i,j \hat{I} \{0,1\}$ the following face equation $X(f,i)(a)=X(g,j)(b)$ holds in $[X_i,X(f,\mathbf{bin}(k))]_0^n$. Therefore, no straightforward 'dimensional continuity' can be maintained in $|X|$. Obviously there exist an embedding $|X|$ into R^{n+1}, but there is no guarantee $|X| \hat{I}$ R^n. For every $|X| \hat{I}$ R^n the following condition holds: for every $a \hat{I} X_i, 0£i<n$ the exist unique $f \hat{I} [i]^{[i+1]}$ such that $a\in\cup_{i=0}^1 \mathrm{Im}(X(f,i))$. Spaces for which the above property also holds include torai, cylinders and spaces that are received from those by surgery and are usually called cylinder- or torus-like spaces. We call cubic spaces for which the aforementioned property holds coordinate preserving cubic spaces (CPC-spaces below). Obviously, the example 3.2.b. belongs to the class of CPC-spaces. Moreover, all dynamical systems that originate from the queuing model of section 2.1. must belong to this class because the state space of these systems is constructed by surgery on the $\boldsymbol{T}^n$. The state spaces of these systems are created by application the following two operation to $\boldsymbol{T}^n$:

- Cutting out cubic subsets of dimensions *i£n* when a subset corresponds to parallel execution of calls within a server while the latter is not capable of required degree of parallelism;
- Sewing in a cubic subset that covers a hole created by cutting operation. These subsets are of the same dimension as a hole and they reflect the scheduling algorithm within a server.

Apparently, application of an arbitrary sequence of the aforementioned operations result in torus-like or cylinder-like space. Therefore, we can say that for our purposes it is enough to consider dynamic systems on CPC-spaces.

4. From dynamics on coordinate preserving cubic spaces to model checking: building Kripke structure.

4.1. Directed graph of dynamical system on CPC-space. In this subsection our goal is to build such a directed graph from a cubic structure that it will allow us to study properties of trajectories of dynamical systems by studying corresponding path in the graph. Let $[X_i, X(f,\mathbf{bin}(k))]$ be a cubic structure of a CPC-space $|X|$, $(\boldsymbol{V}, |X|)$ be a dynamic system defined on $|X|$ (here $\boldsymbol{V}$ denote a vector bundle on $|X|$), and let m be the maximal nonempty grading of this cubic structure (X_m¹Ø, for all $k>m$ X_k=Ø).

<u>Algorithm 4.1.</u> The following algorithm allows as to build a desired graph $G(\boldsymbol{V}, |X|)$:

1. a. Define the initial set of vertices of the graph under construction to be elements of X_m, $V^m=X_m$. Also define a constant map gr_m:$V^m \rightarrow N$::a®m.

   b. Define the initial set of edges E^{m-1} as a set of pairs (s,r)$\in E^{m-1}$ that satisfy the following conditions: if s,r are the elements of X_m such that they have a face l of dimension m-1 in common, the aforementioned face l is a final face in some coordinate i for an element s and it is also an initial face in the same dimension for an element r, and the vector field transversal to a border l then (s,r)$\in E^{m-1}$ i.e., (s,r)$\in E^{m-1}$ if for every $y_1=f(t_0)\in l\times C^{m-1}$, $y_0=f(t_0\text{-}0)\in s$´C^m, $y_2=f(t_0\text{-}0)\in r$´C^m holds $dx_i/dt|_{y0}>0$, $dx_i/dt|_{y1}>0$, $dx_i/dt|_{y2}>0$ then (s,r)$\in E^{m-1}$. Also define a map gr^{m-1}:$E^{m-1}\rightarrow N$::a®m-1. Apparently, from the construction it follows that (s,r)$\in E^{m-1}$ if and only if ther exist corresponding cubes of dimension m in $|X|$ that are traversed by trajectories with certain initial conditions (for example, by definition of dynamic system on coordinate preserving cubic space for every $y\in l\times C^{m-1}$ there must exist $x=f(0)\in s\times C^m$, $z\in s\times C^m$ and times t_0, t_1 ($t_0<t_1$) such that $y=f(t_0)\in l\times C^m$), and $z=f(t_1)\in r\times C^m$;

2. From the sets V^i and E^{i-1}, i£m build the set V^{i-1} as follows:

   a. i. If s,r are the elements of V^i (and therefore elements of X_i) such that they have a face l of dimension i-1 in common such that the face l is a final face of dimension i-1 in some coordinate k£i for both elements r and s, and the vector field inside cubes r and s is transversal to the border l i.e., if l for every $z=f(t_0)\in l\times C^{i-1}$, $y=f(t_0\text{-}0)\in s$´C^i holds $dx_k/dt|_y>0$ and $z=f(t_0)\in l\times C^{i-1}$, $y=f(t_0\text{-}0)\in r$´C^i holds $dx_k/dt|_y>0$. Go to 2.b.i.;

   ii. If s,r are the elements of V^i (and therefore elements of X_i) such that they have a face l of dimension i-1 in common, the face l is the final face of dimension i-1 in some coordinate k£i for element s, face l is the initial face in the same coordinate k for element r, and the vector field inside cube s is transversal to the border l (i.e., for every $z=f(t_0)\in l\times C^{i-1}$, $y=f(t_0\text{-}0)\in s$´C^i holds $dx_k/dt|_y>0$) while the vector field inside cube r is collinear with l (for $z=f(0)\in l\times C^{i-1}$ holds $dx_k/dt|_z=0$ and $y=f(0)\in s$´C^i holds $dx_k/dt|_y=0$) then l belongs to V^{i-1}. Go to 2.b.ii.

   iii. If there exist an element s of V^i that have a face l of dimension i-1 in k'th coordinate that is a boundary (this face is not a face for any other element of V^i) and the vector field inside cube s is transversal to the boundary l (i.e., for every $z=f(t_0)\in l\times C^{i-1}$, $y=f(t_0\text{-}0)\in s$´C^i holds $dx_k/dt|_y>0$) then l belongs to V^{i-1}. Go to 2.b.iii.

   iv. If no elements satisfy conditions of 2.a.i., 2.a.ii. or 2.a.iii. and V^{i-1} is empty then let $V=\cup_{i=1}^{m}V^i$, $gr=\cup_{i=0}^{m}gr_i$: $V\rightarrow N$, $E=\cup_{i=0}^{m-1}E^i$, <u>gr</u>$=\cup_{i=0}^{m}gr^i$:$E\rightarrow N$. Stop. Graph building process is finished.
   If no elements satisfy conditions of 2.a.i., 2.a.ii. or 2.a.iii. and V^{i-1} is not empty then go to 3;

   b. i. If the last element was added to V^{i-1} using the case 2.a.i. then edges (s,l) and (r,l) are added to E^{i-1} ($E^{i-1}=E^{i-1}\cup(s,l)\cup(r,l)$) and the grading map ($gr^{i-1}$:$E^{i-1}\rightarrow N$) is extended to these elements. Go to 2.a. Apparently, from the construction it follows that $(x,y)\in E^{i-2}$ if and only if in $|X|$ there exist cubes of dimensions i and i-1 that are traversed by trajectories with given initial conditions i.e., by definition of cubic space and conditions of 2.a.i. for any $y\in l\times C^{i-1}$ there must exist $x=f(0)\in s\times C^i$, $z=f(0)\in s\times C^i$ and times t_0, t_1 such that $y=f(t_0)|_{x=\phi(0)}$ and $y=f(t_1)|_{z=f(0)}$ where $f(t)$ is the integral curve of the vector field $\boldsymbol{V}$;

   ii. If the last element was added to V^{i-1} using the case 2.a.ii. then edge (s,l) is added to E^{i-1} ($E^{i-1}=E^{i-2}\cup(s,l)$) and the grading map (gr^{i-2}:$E^{i-2}\rightarrow N$) is extended to the new element of E^{i-2}. Go to 2.a. Note that the construction here is such that $(x,y)\in E^{i-2}$ if and only if in $|X|$ there exist cubes of dimensions i and i-1 that are traversed by trajectories with given initial condition i.e., by definition of cubic space and conditions of 2.a.ii. for any $y\in l\times C^{i-1}$ there must exist $x=f(0)\in s\times C^i$ and a time t_0 such that $y=f(t_0)|_{x=\phi(0)}$ where $f(t)$ is the integral curve of the vector field $\boldsymbol{V}$;

   iii. If the last element was added to V^{i-1} using the case 2.a.iii. then the edge (s,l) is added to E^{i-1} ($E^{i-1}=E^{i-1}\cup(s,l)$) and the grading map (gr^{i-1}:$E^{i-1}\rightarrow N$) is extended to this elements. Go to 2.a.
   The remark from 2.b.ii. is true here as well i.e., $(x,y)\in E^{i-1}$ if and only if in $|X|$ there exist cubes of

dimensions i and $i-1$ that are traversed by trajectories with given initial condition. In other words, by definition of cubic space and conditions of 2.a.ii. for any $y\in l\times C^{i-1}$ there exist $x=f(0)\in s\times C^i$ and a time t_0 such that $y=f(t_0)|_{x=\phi(0)}$ where $f(t)$ is the integral curve of the vector field $\boldsymbol{V}$.

3 For all nodes $s\in V^{i-1}$ do the following:

a. For every coordinate $k£i-1$ such that for a $y\in s´\underline{C}^{i-1}$ holds that $dx_k/dt|_y>0$ find the set of all elements of higher or equal dimension such that y has a face of dimension $i-2$ in coordinate k in common with x, and this common face is final for x and initial in all additional dimensions for the other element, i.e., $Y=\{y|y\in V^j, j\,^3 i-1, X(f^k,1)(x)=X(g_1{}^x...{}^x g_{j-i},0i_1...i_{(i-j-1)})(y)$ where $g_1=f^k$ and $i_l\in\{0,1\}$;

b. Construct subset of Y containing only elements of maximal grading and no faces of these elements by deleting from Y all elements that are faces of other elements of Y;

c. Build a new set Y' by selecting elements $y\in Y$ such that for any $z\in X(f^k,1)(x)´\underline{C}^{i-2}$ and any $z'\in y\times\underline{C}^{gr(y)}$ holds $\forall i\ [dx_i/dt|_z=dx_i/dt|_{z'}]$;

d. Let $k=\max\{gr(y)|y\in Y\}$. Choose the from Y' an element y_0 such that y_0 is an element on which the $\min\{i_1...i_{(i-j-1)}|X(f^k,1)(x)=X(g_1{}^x...{}^x g_{j-i},0i_1...i_{(i-j-1)})(y), gr(y)=k, y\in Y\}$. Add (s,y) to E^{i-2};

e. Define a constant map gr_i:$V^i\rightarrow N::a®i$, define a map $gr^{i-2}:E^{i-2}\rightarrow N::a®i-2$. If $i-1>0$ then set $i=i-1$ and go to 2.a.
Otherwise, let $V=\cup_{i=1}^{m}V^i$, $gr=\cup_{i=0}^{m}gr_i$: $V\rightarrow N$, $E=\cup_{i=0}^{m-1}E^i$, $\underline{gr}=\cup_{i=0}^{m}gr^i:E\rightarrow N$. Stop. Graph building process is finished.

Note that in the step 3 the $(s,y)\in E^{i-2}$ if and only if s is lower grading than y (the same is true of dimensions of corresponding cubes in $|X|$), the flow in the s-cube is transversal to the flow in its corner and the flow in the corner coincide with the flow in y-cube thus insuring that there exist a trajectory with given initial conditions that traverses s-cube and then y-cube in $|X|$, i.e., for any $z\in x´C^{i-1}$, there exist $z'\in X(f^k,1)(x)´\underline{C}^{i-2}$ and $z''\in y\times\underline{C}^{gr(y)}$ and times t_0, t_1 such that $z=f(0)$, $z'=f(t_0)|_{z=f(0)}$, $z''=f(t_1)|_{z=f(0)}$ where $f(t)$ is the integral curve of the vector field $\boldsymbol{V}$. Obviously y is the element of highest grading that lies on this trajectory immediately next to s.

For a trajectory $f(t)$ in $(\boldsymbol{V},|X|)$ let us define an ~ relation if the following holds $(s,x_1)\sim(s,x_2)$ if $x_1,x_2\in\underline{C}^{gr(s)}$. Let us also define the projection $\pi:|X|®\cup_{i=0}^{m}X_i::(s,x)\rightarrow s$. Therefore one can associate to each trajectory a sequence $\pi(f(t)/\sim)$. Note that $\pi(f(t)/\sim)$ is an infinite sequence when $t®¥$. For every elements $a,b,c\in\cup_{i=0}^{m}X_i$ let us define sequence reduction map $red(abc)\rightarrow ac$ if for $z\in a\times\underline{C}^{gr(a)}$, $z\in b\times\underline{C}^{gr(b)}$, $z\in c\times\underline{C}^{gr(c)}$ holds $\forall i\ [dx_i/dt|_z=dx_i/dt|_{z'}=dx_i/dt|_{z''}]$ and $gr(a)=gr(c)=gr(b)+1$; $red(abc)=abc$ otherwise. The reduction map can be extended to sequence of arbitrary length by left to right application rule. We call $red(\pi(f(t)/\sim)$ symbolic trace of the trajectory $f(t)$.

__Proposition 4.2.__ For a trajectory $f(t)$ of the dynamical system $(\boldsymbol{V},|X|)$ on CPC-space $|X|$ its symbolic trace $red(\pi(f(t)/\sim)$ is the run in $G(\boldsymbol{V},|X|)$.

Proof: straightforward, holds by construction.

Therefore, if one can prove that all runs in $G(\boldsymbol{V},|X|)$ have certain property then symbolic traces of trajectories in $(\boldsymbol{V},|X|)$ also have this property.

4.2. Example 3.2.b. (continued). It is easily verifiable that the graph for this example is given by the following sets $V=V^2=\{s_{00}, s_{01}, s_{10}, s_{11}, s_{11}{}^1\}$, $E=E^1=\{(s_{00}, s_{01}), (s_{00}, s_{10}), (s_{01}, s_{00}), (s_{10}, s_{00}), (s_{01}, s_{11}), (s_{11}, s_{01}), (s_{10}, s_{11}{}^1), (s_{11}{}^1, s_{10})\}$. See fig. 4.1.☐

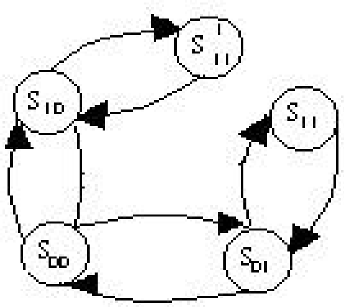


Fig 4.1.

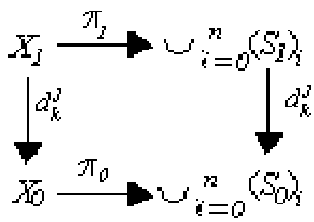
4.2 Marking a directed graph. Traditionally in analysis of client server protocols researchers are interested in:

- Cumulative characteristics of the system such as delay probability, average queue length, etc. (see for example [ICT, 97];
- Individual history of a calls and cooperative history of all calls in the system (see [CK+, 93]).

For the purposes of statistical analysis mapping trajectories into runs on directed graph does not help. However, it may be useful in investigation of a cooperative history of the system. The problem is that we have no idea in what state is an individual call when a system is in vertex x of a $G(\boldsymbol{V}, |X|)$. To reconstruct individual behavior of calls we need to go back to $|X|$. Apparently, each call by itself is a one-dimensional cubic space of the type given on fig. 2.2. Let us denote these dynamical systems $(\mathbf{ST}, \mathbf{S})_i$. Recall that state spaces that result from queuing networks of the type described in section 2. are CPC-spaces. Therefore, there exist a natural projection p_i of $|X|$ onto its i'th coordinate. Let now $|X_p|=\Pi_{i=0}^{n}p_i(|X|)$ and $<p_1,\ldots,p_n>: |X|\rightarrow|X_p|$. Apparently, $|X_p|$ has the cubic structure $[X_p,X_p(f,\mathbf{bin}(k))]$ induced by $<p_1,\ldots,p_n>$, and a corresponding map of cubic structures is denoted $\underline{<p_1,\ldots,p_n>}:[X,X(f,\mathbf{bin}(k))]\rightarrow[X_p,X_p(f,\mathbf{bin}(k))]$. The map $\underline{<p_1,\ldots,p_n>}$ induces surjection

p_1:$\cup_{i=0}^{n}(X_1)$ $\rightarrow S_1$, p_0:$\cup_{i=0}^{n}(X_0)_i$ $\rightarrow S_0$. These surjections map elements of X_1 onto $\cup_{i=0}^{n}(S_1)_i$ and elements of X_0 into $\cup_i^{n}(S_0)_i$.in the following way:

- The map p is determined by maps from $\{f:[n-1]\rightarrow[n]\}$. Informally it maps 1-graded faces of all $n-1$-graded i-faces of the same element of X_n into the same element of $\cup_i^{n}(S_1)_i$. Moreover, by the properties of sewing maps if $s,v \ni X_n$ and they have $n-1$-dimensional i-face in common then all of the 1-graded faces of all $n-1$-graded i-faces of s and v are mapped into the same element of $\cup_i^{n}(S_1)_i$.
- The map p_0 is completed to make up the following commutative squares (here $k,j \ni \{0,1\}$):



Therefore, for an element $s\,' X_m$ every map $X(f,x_0x_1...x_{m-1})$:$X_m \rightarrow X_1$:: s ® $X(f,x_0x_1...x_{m-1})(s)$ can be extended to

$$p_1\cdot X(f,x_0x_1...x_{m-1}):: X_m \rightarrow \cup_i^{n}(S_1)_1$$

mapping s to individual state of i'th call. The conditions on $\leq p_1,\ldots,p_n \geq$ guarantees that this map is well defined. Therefore, the set of all maps of the type $p_1\cdot X(f,x_0x_1...x_{m-1})$ map s to individual states of all calls.

Let us consider again the collection of CPC dynamical systems $(\mathbf{ST, S})_i$. We introduce the language L of Propositional Logic with a standard signature and the set of propositional variables P= $\cup_i^{n}(S_1)_i$. Then we can define a map on from φ:[X,X(f,**bin**(k))] $\rightarrow$L as follows:

For s$\ni X_m$ let VAR be the set of all variables corresponding to individual states to which s is mapped by $p_1\cdot X(f,x_0x_1...x_{m-1})$, i.e., VAR={x|x$\ni$P, x=$p_1\cdot X(f,x_0x_1...x_m$-1)($s$) for some pair ($f,x_0x_1...x_m$-1)}. Then let $\varphi(s)=(\&_{Z\ni VAR}Z)\&(\&_{Z\ni VAR}(\sim Z))$.

For a CPC dynamical system ($\boldsymbol{V}$,|X|), every element of V in its graph G ($\boldsymbol{V}$,|X|) =(V,E) is also the elements of X_i. Therefore, the restriction of φ to V (φ|V) is well defined. Hence, we completed marking of the graph. We denote the marked graph of CPC dynamical system ($\boldsymbol{V}$,|X|) by a pair <φ, G ($\boldsymbol{V}$,|X|)>. This completes the construction of a marked graph that can be used as a model (Kripke structure) in model checking of temporal formulae.

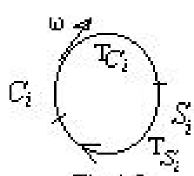


Fig.4.2.

4.3. Example 3.2.b. (continued). In this example a route of each individual client is C_i,S_i with times T_{C_i}, T_{S_i} correspondingly. Corresponding dynamical system (**ST**,**S**) is given on fig. 4.2. Marked graph <φ_i,G ($\mathbf{ST}_i$,$\mathbf{S}_i$)>, $i\,'\{0,1\}$ has V={C_i,S_i}, E={(C_i,S_i), (S_i,C_i)}, L={C_i,S_i }, $\varphi_i(C_i)$= C_i&~S_i, $\varphi_i(S_i)$= S_i&~C_i (see fig. 4.3.) . Therefore, the associate set of variables for the language L is {C_0, C_1, S_0, S_1}. For a 2-dimensional LIFO dynamical system

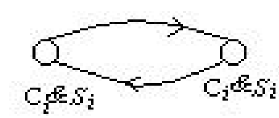


Fig.4.3.

($\boldsymbol{V}_{LIFO}$, |LIFO|) the marked graph <φ_{LIFO}, G ($\boldsymbol{V}_{LIFO}$, |LIFO|))> is constructed from unmarked graph (Fig. 4.1.) by setting

$\varphi(S_{00})$= C_0&C_1&~S_0&~S_1, $\varphi(S_{01})$= ~C_0&C_1&S_0&~S_1,
$\varphi(S_{10})$= C_0&~C_1&~S_0&S_1, $\varphi(S_{11})$= ~C_0&~C_1&S_0&S_1,
$\varphi(S_{11}^{1})$= ~C_0&~C_1&S_0&S_1 (see fig. 4.4.).

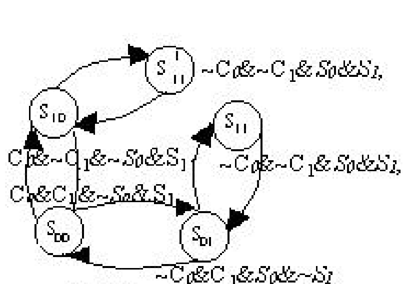


Fig 4.4

5. Discussion

In section 1 we outlined the method for checking properties of client server systems that includes the following steps:

1. Formalization of a property of interest;
2. Construction of a marked graph from a dynamical system;
3. Model-checking of a property

This method assumes that a model in the form of dynamical system is already known or (at least) that it is well known how to construct the dynamical system out of the queuing network that describes the system. Though this construction cannot be automated, it is usually straightforward. Construction of queuing network for the system is simple because this model corresponds to what one can see in the real system; construction of dynamical systems from queuing network is usually not difficult because phase space representation of queuing networks is intuitively appealing and serves as a good illustration to systems functioning. We designed an algorithm that given a CPC dynamical system creates a Kripke structure to be used in step 3 of this method. The step 3 is fully automatic and verification tools are readily available.

We must caution the reader that this method is only good for those properties that are true for all systems that have differ only in time parameters. Therefore, properties that cannot be checked on symbolic traces of orbits cannot be checked by this method. Yet we believe that properties of specific systems may be included into this method and we plan to address this issue in future. Especially promising seems to be the approach suggested in [SS, 98].

To complete the investigation of the proposed method complexity of the algorithm presented must be analyzed. Because of space limitations we were unable to present the analysis in full. We claim that it lies in PSPACE. We plan to address this matter elsewhere.

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